

▲ **Figure 2.1** Typical multicamera setup for a 3-D kinematic analysis.

3-D coordinates from 2-D camera coordinates, called the *direct linear transformation* (DLT) method (Abdel-Azis and Karara 1971), assumes a linear relationship between the 2-D camera coordinates of a marker and the 3-D laboratory coordinates of the same marker. The DLT method is described in more detail in papers Marzan and Karrara (1975) and is not elaborated here.

COORDINATE SYSTEMS AND ASSUMPTION OF RIGID SEGMENTS

In this chapter, we define a number of Cartesian coordinate systems required for a 3-D analysis. These are referred to as a *global or laboratory coordinate system* (GCS), a *segment or local coordinate system* (LCS), and a *force platform coordinate system* (FCS).

For this chapter, a biomechanical model is a collection of rigid segments. A segment's interaction with other segments is described by joint constraints permitting zero to six degrees of freedom, and subject-specific scaling is defined using palpable anatomical landmarks.

These rigid segments represent skeletal structures, which are not always represented ideally as rigid segments. For example, some segments, like the foot or the torso, often have one segment representing several bones. It is incorrect to assume that skeletal structures are rigid, but it makes the mathematics more palatable. The assumption of rigidity also aids the establishment of an LCS.

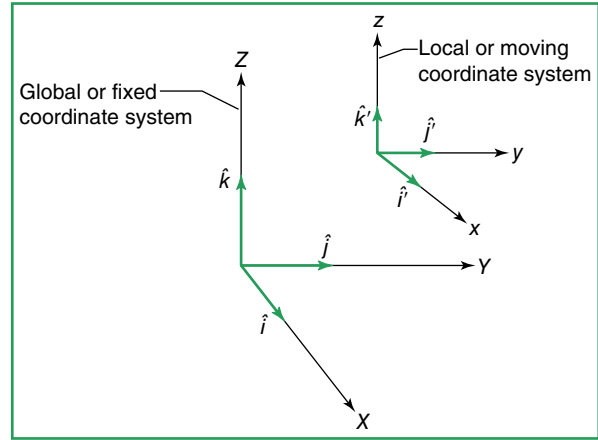
Global or Laboratory Coordinate System

The GCS refers to the capture volume in which we represent the 3-D space of the motion-capture system (also referred to as the *inertial reference system*). Recorded data are resolved into this fixed coordinate system. In this chapter, the GCS is designated using uppercase letters with the arbitrary designation of XYZ. The Y-axis is nominally directed anteriorly, the Z-axis is directly superiorly, and the X-axis is perpendicular to the other two axes. Because the subject may move anywhere in the data collection volume, only the vertical direction needs to be defined carefully, and that is only so we have a convenient representation of the gravity vector. The unit vectors for the GCS are \hat{i} , \hat{j} , \hat{k} (see figure 2.2). In this chapter, and in the biomechanics literature, the GCS is a right-handed orthogonal system with an origin that is fixed in the laboratory. Note that a coordinate system is right-handed if and only if

$$\hat{k} = \hat{i} \times \hat{j} \text{ and } \hat{i} \cdot \hat{j} = 0 \quad (2.1)$$

Segment or Local Coordinate System

A mathematically convenient consequence of the assumption of rigidity is that in the context of kinematics, each segment is defined completely by an LCS fixed in the segment; as the segment moves, the LCS moves correspondingly. Like the GCS, the LCS is right-handed and orthogonal. In this chapter, the LCS is designated in lowercase letters x , y , z and unit vectors \hat{i}' , \hat{j}' , and \hat{k}' , respectively. In this chapter, the LCS is oriented such that the y -axis points anteriorly, the z -axis points axially (typically vertically), and the x -axis is perpendicular to the plane of the other two axes with its direction defined by the right-hand rule. Thus, on the right side the x -axis is directed from medial to lateral, whereas on the left side it is directed from lateral to medial. The orientation of the LCS with respect to the GCS defines the orientation of the body or segment in the GCS, and it changes as the body or segment moves through the 3-D space (see figure 2.2).



▲ **Figure 2.2** The global or fixed coordinate system, XYZ, with unit vectors, \hat{i} , \hat{j} , and \hat{k} and the local or moving coordinate system, xyz, and its unit vectors, \hat{i}' , \hat{j}' , and \hat{k}' .

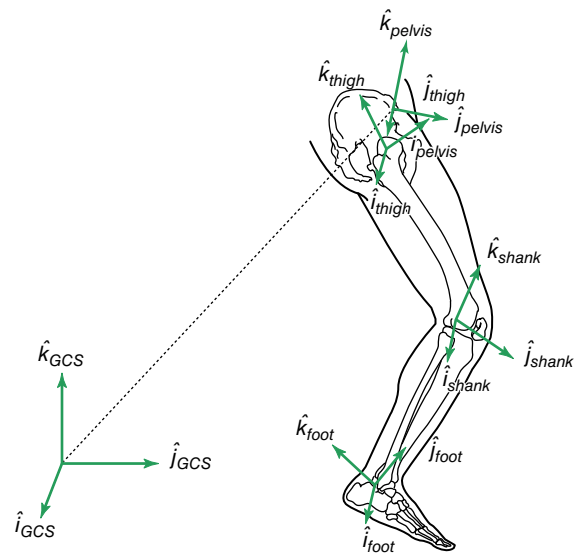
TRANSFORMATIONS BETWEEN COORDINATE SYSTEMS

We have identified two types of coordinate systems (GCS and LCS) that exist in the same 3-D motion-capture volume. The descriptions of a rigid segment moving in space in different coordinate systems can be related by means of a *transformation* between the coordinate systems (see figure 2.3). A transformation allows one to convert coordinates expressed in one coordinate system to those expressed in another coordinate system. In other words, we can look at the same location in different ways based on which coordinate system we are using. At first glance this may seem redundant because we have not added new information by describing the same point in different ways. It is, however, convenient because objects move in the GCS but attributes of a segment, such as an anatomical landmark (e.g., segment endpoint), are constant in the LCS. We generally refer to transformations as linear or rotational.

Linear Transformation

In figure 2.4, a point is described by the vector \vec{P}' in the LCS and by \vec{P} in the GCS. The linear transformation between the LCS and the GCS can be defined by a vector \vec{O} , which specifies the origin of the LCS relative to the GCS. The components of \vec{O} can be written as a column matrix in the form

$$\vec{O} = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} \quad (2.2)$$



▲ **Figure 2.3** The global coordinate system and the local coordinate systems of the right-side lower extremity.

If we assume no rotation of the LCS relative to the GCS, converting the coordinates of a point \vec{P}' in LCS to \vec{P} in GCS can be expressed as

$$\vec{P} = \vec{P}' + \vec{O} \quad (2.3)$$

$$\text{or } \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} P'_x \\ P'_y \\ P'_z \end{bmatrix} + \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} \quad (2.4)$$

Conversely, conversion of the coordinates of a point \vec{P} in GCS to \vec{P}' in LCS is expressed as

$$\vec{P}' = \vec{P} - \vec{O} \quad (2.5)$$

Rotational Transformation

If we assume no translation of the LCS relative to the GCS, converting the coordinates of a point \vec{P} in GCS to \vec{P}' in LCS can be expressed as

$$\vec{P}' = R\vec{P} \quad (2.6)$$

where R is a matrix made up of orthogonal unit vectors (orthonormal matrix) that rotates the GCS about its origin, bringing it into alignment with the LCS. Conversely, converting the coordinates of a point in the LCS to a point in the GCS can be accomplished by

$$\vec{P} = R'\vec{P}' \quad (2.7)$$

where R' is the inverse (and the transpose) of R . In this chapter we consistently use R as the transformation from GCS to LCS and R' as the transformation from LCS to GCS.

Consider the LCS unit vectors $\hat{i}', \hat{j}', \hat{k}'$ expressed in the GCS. The rotation matrix from GCS to LCS is as follows:

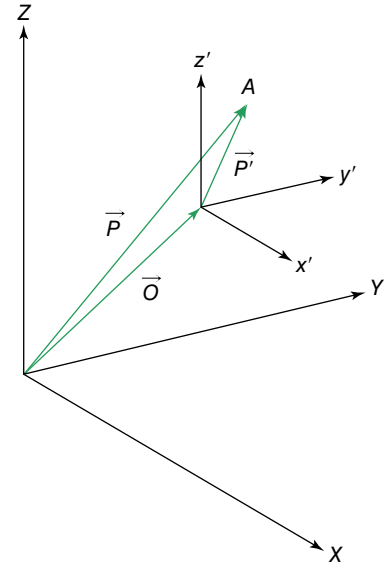
$$R = \begin{bmatrix} \hat{i}'_x & \hat{i}'_y & \hat{i}'_z \\ \hat{j}'_x & \hat{j}'_y & \hat{j}'_z \\ \hat{k}'_x & \hat{k}'_y & \hat{k}'_z \end{bmatrix} \quad (2.8)$$

If we consider translation and rotation of the LCS relative to the GCS, converting the coordinates of a point \vec{P} in GCS to \vec{P}' in LCS can be expressed as

$$\vec{P}' = R(\vec{P} - \vec{O}) \quad (2.9)$$

Conversely, converting the coordinates of a point \vec{P}' in the LCS to point \vec{P} in GCS can be accomplished by

$$\vec{P} = R'\vec{P}' + \vec{O} \quad (2.10)$$



▲ **Figure 2.4** The point A is defined by vector \vec{P} in XYZ, whereas the same point is defined by vector \vec{P}' in $x'y'z'$. The linear transformation between $x'y'z'$ and XYZ can be defined by a vector \vec{O} , which specifies the origin of the LCS relative to the GCS.

DEFINING THE SEGMENT LCS FOR THE LOWER EXTREMITY

In this chapter we use three noncollinear points to define a segment's LCS. *Noncollinear* means that the points are not aligned or in a straight line. The method presented is consistent with most models presented in the biomechanics literature. In this chapter, the LCS is described for the right-side segments of a lower-extremity model consisting of a pelvis, thigh, shank, and foot segments (the left side uses

a similar derivation). The LCS of each segment is created based on a standing calibration trial and on palpable anatomical landmarks (see table 2.1). It is important to note that both the tracking and calibration markers are captured at the same time. However, in figure 2.5, only the calibration markers are shown.

Table 2.1 Abbreviations for Calibration Markers

Right	Description	Left	Description
\vec{P}_{RASIS}	Right anterior-superior iliac spine	\vec{P}_{LASIS}	Left anterior-superior iliac spine
\vec{P}_{RPSIS}	Right posterior-superior iliac spine	\vec{P}_{LPSIS}	Left posterior-superior iliac spine
\vec{P}_{RLK}	Right lateral femoral epicondyle	\vec{P}_{LLK}	Left lateral femoral epicondyle
\vec{P}_{RMK}	Right medial femoral epicondyle	\vec{P}_{LMK}	Left medial femoral epicondyle
\vec{P}_{RLA}	Right lateral malleolus	\vec{P}_{LLA}	Left lateral malleolus
\vec{P}_{RMT5}	Right fifth metatarsal head	\vec{P}_{LMT5}	Left fifth metatarsal head
\vec{P}_{RMT1}	Right first metatarsal head	\vec{P}_{LMT1}	Left first metatarsal head
\vec{P}_{RHEEL}	Right heel	\vec{P}_{LHEEL}	Left heel
\vec{P}_{RTOE}	Right toe	\vec{P}_{LTOE}	Left toe

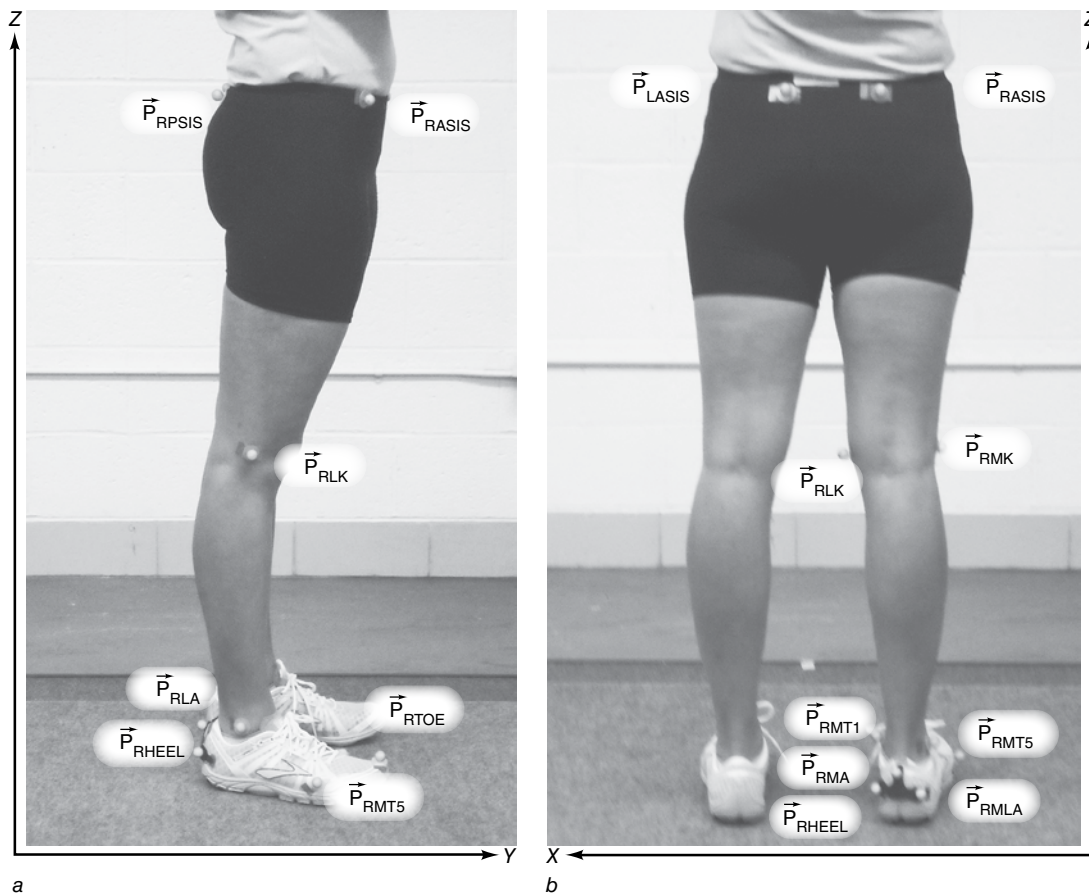


Figure 2.5 Right-side marker configuration of the calibration markers used in this chapter to compute the local coordinate system of each segment: (a) sagittal plane; (b) frontal plane. The \vec{P}_{LPSIS} and \vec{P}_{LASIS} are not seen in part a but are necessary for calibration. The \vec{P}_{RPSIS} , \vec{P}_{LPSIS} , and \vec{P}_{RTOE} are not seen in part b but are necessary for calibration. Note that the tracking markers on the thigh, shank, and foot are necessary in the calibration trial to associate these with the LCS of each segment.

Pelvis Segment LCS

Markers are placed on the following palpable bony landmarks: right and left anterior-superior iliac spine (\vec{P}_{RASIS} , \vec{P}_{LASIS}) and right and left posterior-superior iliac spine (\vec{P}_{RPSIS} , \vec{P}_{LPSIS}) (figure 2.6). The origin of the LCS is midway between \vec{P}_{RASIS} and \vec{P}_{LASIS} and can be calculated as follows:

$$\vec{O}_{PELVIS} = 0.5 * (\vec{P}_{RASIS} + \vec{P}_{LASIS}) \quad (2.11)$$

To create the x -component (or lateral direction) of the pelvis, a unit vector \hat{i}' is defined by subtracting \vec{O}_{PELVIS} from \vec{P}_{RASIS} and dividing by the norm of the vector:

$$\hat{i}' = \frac{\vec{P}_{RASIS} - \vec{O}_{PELVIS}}{|\vec{P}_{RASIS} - \vec{O}_{PELVIS}|} \quad (2.12)$$

Next we create a unit vector from the midpoint of \vec{P}_{RPSIS} and \vec{P}_{LPSIS} to \vec{O}_{PELVIS} :

$$\hat{v} = \frac{\vec{O}_{PELVIS} - 0.5 * (\vec{P}_{RPSIS} + \vec{P}_{LPSIS})}{|\vec{O}_{PELVIS} - 0.5 * (\vec{P}_{RPSIS} + \vec{P}_{LPSIS})|} \quad (2.13)$$

A unit vector normal to the plane (in the superior direction) containing \hat{i}' and \hat{v} is computed from a cross product:

$$\hat{k}' = \hat{i}' \times \hat{v} \quad (2.14)$$

Note that the order in which the vectors \hat{i}' and \hat{v} are crossed to produce a superiorly directed unit vector is determined by the right-hand rule. At this point we have defined the lateral direction and the superior direction. The anterior unit is created from the cross product

$$\hat{j}' = \hat{k}' \times \hat{i}' \quad (2.15)$$

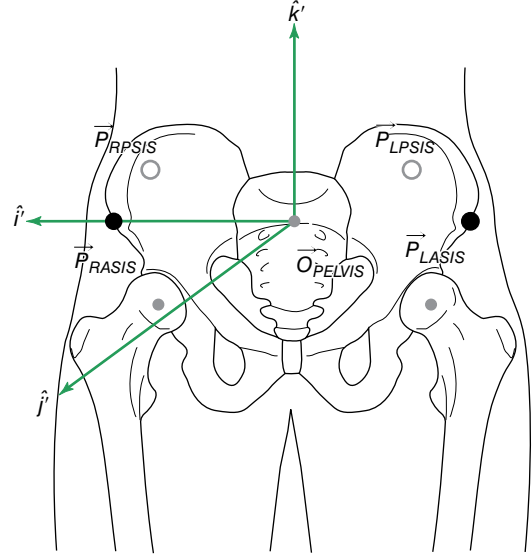
The rotation matrix describing the orientation of the pelvis, which will be used in later calculations, is constructed from the unit vectors as

$$\mathbf{R}_{PELVIS} = \begin{bmatrix} \hat{i}'_x & \hat{i}'_y & \hat{i}'_z \\ \hat{j}'_x & \hat{j}'_y & \hat{j}'_z \\ \hat{k}'_x & \hat{k}'_y & \hat{k}'_z \end{bmatrix} \quad (2.16)$$

Thigh Segment LCS

The thigh is defined by one virtual location and two marker locations (figure 2.7). The proximal end (and origin) of the thigh is coincident with the location of a virtual hip joint center. A number of studies have described regression equations for estimating the location of the hip joint center in the pelvis LCS (Andriacchi et al. 1980; Bell et al. 1990; Davis et al. 1991; Kirkwood et al. 1999). In this chapter we use equations derived from Bell and colleagues (1989) to compute a landmark that represents the hip joint center \vec{P}_{RHIP} in the pelvis LCS:

$$\vec{P}_{RHIP} = \begin{bmatrix} 0.36 * |\vec{P}_{RASIS} - \vec{P}_{LASIS}| \\ -0.19 * |\vec{P}_{RASIS} - \vec{P}_{LASIS}| \\ -0.30 * |\vec{P}_{RASIS} - \vec{P}_{LASIS}| \end{bmatrix} \quad (2.17)$$



▲ **Figure 2.6** The origin of the pelvis LCS (\vec{O}_{PELVIS}) is midway between the right and left anterior-superior iliac spines. The right and left anterior-superior iliac spines (\vec{P}_{RASIS} and \vec{P}_{LASIS}) and the posterior-superior iliac spines (\vec{P}_{RPSIS} and \vec{P}_{LPSIS}) can be used to derive the pelvis LCS.

We can transform the location of the hip joint from the pelvis LCS to the GCS as follows:

$$\vec{O}_{RTHIGH} = \vec{P}_{RHIP} = R'_{PELVIS} * \vec{P}_{RHIP} + \vec{O}_{PELVIS} \quad (2.18)$$

To develop the thigh LCS, a superior unit vector is created along an axis passing from the distal end (midpoint between the femoral epicondyles \vec{P}_{RLK} and \vec{P}_{RMK}) to the origin (\vec{O}_{RTHIGH}) as follows:

$$\hat{k}' = \frac{\vec{O}_{RTHIGH} - 0.5 * (\vec{P}_{RLK} + \vec{P}_{RMK})}{|\vec{O}_{RTHIGH} - 0.5 * (\vec{P}_{RLK} + \vec{P}_{RMK})|} \quad (2.19)$$

We then create a unit vector passing from the medial to the lateral femoral epicondyle:

$$\hat{v} = \frac{(\vec{P}_{RLK} - \vec{P}_{RMK})}{|\vec{P}_{RLK} - \vec{P}_{RMK}|} \quad (2.20)$$

The anterior unit vector is determined from the cross product of the \hat{k}' and \hat{v} vectors as follows:

$$\hat{j}' = \hat{k}' \times \hat{v} \quad (2.21)$$

Care should be taken in the placement of the knee markers. The lateral marker is placed at the most lateral aspect of the femoral epicondyle. The medial marker should be located so that the lateral and medial knee markers and the hip joint define the frontal plane of the thigh.

Last, the unit vector in the lateral direction is formed from the cross product:

$$\hat{j}' = \hat{k}' \times \hat{v} \quad (2.22)$$

The rotation matrix describing the orientation of the thigh is constructed from the thigh unit vectors as

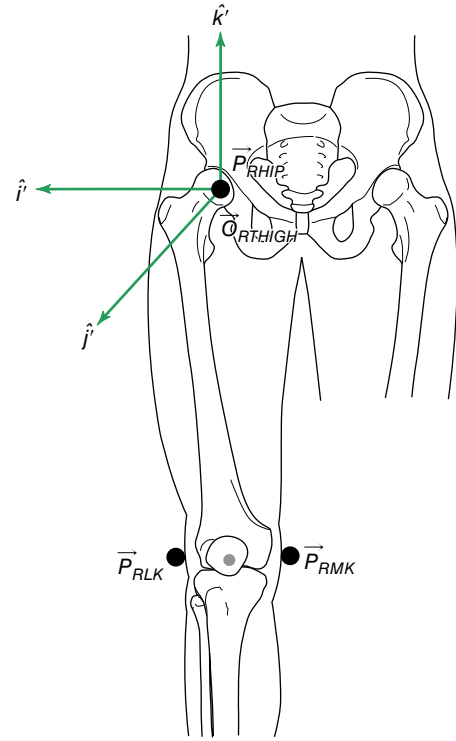
$$R_{RTHIGH} = \begin{bmatrix} \hat{i}'_x & \hat{i}'_y & \hat{i}'_z \\ \hat{j}'_x & \hat{j}'_y & \hat{j}'_z \\ \hat{k}'_x & \hat{k}'_y & \hat{k}'_z \end{bmatrix} \quad (2.23)$$

Because the LCS is orthogonal and \hat{k}' is defined explicitly to pass between the segment endpoints, the lateral unit vector \hat{i}' , which is perpendicular to \hat{k}' , is not necessarily parallel to an axis passing between the epicondyles. In other words, the placement of the medial and lateral knee markers does not define the flexion-extension axis.

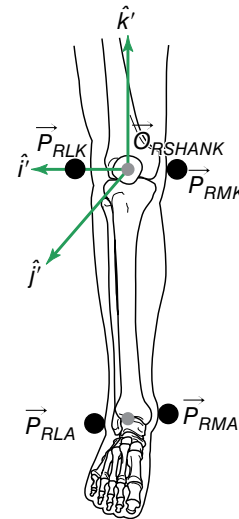
Shank Segment LCS

For the shank or leg segment, the LCS is defined from four palpable landmarks: the lateral and medial malleoli \vec{P}_{RLA} and \vec{P}_{RMA} and the lateral and medial femoral epicondyles, \vec{P}_{RLK} and \vec{P}_{RMK} (figure 2.8). The origin of the LCS is at the midpoint between the femoral epicondyles and can be calculated as

$$\vec{O}_{RSHANK} = 0.5 * (\vec{P}_{RLK} + \vec{P}_{RMK}) \quad (2.24)$$



▲ **Figure 2.7** The origin of the thigh LCS (\vec{O}_{RTHIGH}) is at the hip joint center. The position of hip joint center (\vec{P}_{RHIP}) and the lateral and medial femoral epicondyles (\vec{P}_{RLK} and \vec{P}_{RMK}) can be used to calculate the thigh LCS.



▲ **Figure 2.8** The origin of the shank LCS (\vec{O}_{RSHANK}) is located at the midpoint of the lateral and medial epicondyles (\vec{P}_{RLK} and \vec{P}_{RMK}). The positions of the lateral and medial epicondyles (\vec{P}_{RLK} and \vec{P}_{RMK}) and the midpoint of the lateral and medial malleoli (\vec{P}_{RLA} and \vec{P}_{RMA}) can be used to derive the local coordinate system of a proximal biased shank.